



Contact temperature and wear of composite friction elements during braking

S.J. Matysiak^{a,*}, A.A. Yevtushenko^b, E.G. Ivanyk^c

^a Faculty of Geology, Institute of Hydrogeology and Engineering Geology, University of Warsaw,
Al. Żwirki Wigury 93, 02-089 Warsaw, Poland

^b Faculty of Civil Engineering, Architecture and Environmental Engineering, Technical University of Łódź,
Al. Politechniki 6, 93-590 Łódź, Poland

^c Faculty of Applied Mathematics, Technical University of Lviv, Bandery str.12, 290 601 Lviv, Ukraine

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Abstract

The analytical model for the determination of contact temperature and wear on a working surface of friction brakes is presented. It is assumed that one of the friction element is composed of a periodic two-layered composite and the second element is a homogeneous half-space. In the frictional process, the wear coefficient is linearly dependent on the contact temperature. The influences of composite parameters as well as a parameter characterizing the changing of loading from zero to the nominal value on the distribution of contact temperature and wear during braking is considered. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The paper deals with the problem of contact temperature and wear on a working surface of friction brakes. It is assumed that one of the friction element is composed of a periodic two-layered composite and the second element is a homogeneous half-space. The composite element is continuously sliding with friction on the boundary plane of the half-space. The presented problem can be applied to brakes constituted of two sliding discs (like airplane brakes). In frictional process, the wear coefficient is assumed to be linearly dependent on the contact temperature.

In the papers [1,2] the surface temperature of the friction brake elements is determined as a sum of the surrounding temperature, volumes and mean contact temperature. The mean temperature of nominal contact area is given by solving of the one-dimensional boundary-value problem of heat conduction for two bodies: the layered composite half-space $z \geq 0$ – denoted by index p and a homogeneous disc $z \leq 0$ – (denoted by index d), see [2]. It was assumed that the materials of the

bodies are homogeneous and isotropic. However, in modern systems of braking, some composite materials are used. Especially, structures composed of periodically repeated different layers can be applied.

In this paper, the layered composite half-space is taken into considerations. The approach based on the homogenized model with microlocal parameters for microperiodic two-layered composites [3,4] is applied to an analysis of contact temperature and wear during braking.

The homogenized model constitutes an approximate theory of periodically composite being useful in solving several types of boundary conditions for periodically stratified thermoelastic composites, in which thermal and mechanical continuity conditions on interfaces are satisfied.

The review of papers connected with microlocal modeling of periodic composites is given in [5].

2. Formulation of the problem

The problem of the frictional heating during braking is determined by solutions of the equations of nonstationary heat conduction

* Corresponding author.

Nomenclature	
c^*	ratio, $c^* = c_2/c_1$
c_1, c_2	specific heats of the subsequent layers of composite
$\operatorname{erfc}(\cdot)$	$= 1 - \operatorname{erf}(\cdot)$, erf(\cdot)-error function
f	coefficient of friction
k	thermal diffusivity
K^*	ratio, $K^* = K_2/K_1$
K_1, K_2	conductivities of the subsequent layers of composite
m	coefficient of wear
p	load
p^*	dimensionless load, $p^*(t) = 1 - \exp(-t)$
p_0	maximal load
T	temperature
T^*	dimensionless contact temperature
t	time
τ	dimensionless time, $\tau = t/t_s^0$
τ^*	dimensionless time, $\tau^* = t/t_m$
t_s^0	stopping time in the case of constant pressure during braking
t_s	braking time
t_m	characteristic rise time of the loading
W	initial kinetic energy per unit area
q	rate of the frictional heating
V	sliding speed
V_0	initial sliding speed
z	axial coordinate
h_1, h_2	thicknesses of layers being composite components
h	thickness of fundamental layer, $h = h_1 + h_2$
η	ratio, $\eta = h_1/h$
ρ_1, ρ_2	densities of the subsequent layers of composite
I	wear
<i>Subscripts</i>	
d	homogeneous friction element
p	periodic two-layered friction element

$$\frac{\partial^2 T_i(z, t)}{\partial z^2} = \frac{1}{k_i} \frac{\partial T_i(z, t)}{\partial t}, \quad 0 \leq t \leq t_s, \quad (1)$$

$$z > 0 \quad \text{for } i = p,$$

$$z < 0 \quad \text{for } i = d,$$

with the initial conditions

$$T_i(z, 0) = 0, \quad i = p, d \quad (2)$$

and the coupling conditions

$$T_p(0^+, t) = T_d(0^-, t) \equiv T(t), \quad 0 \leq t \leq t_s, \quad (3)$$

$$-K_p \left. \frac{\partial T_p(z, t)}{\partial z} \right|_{z=0^+} + K_d \left. \frac{\partial T_d(z, t)}{\partial z} \right|_{z=0^-} = q(t), \quad (4)$$

$$0 \leq t \leq t_s$$

as well as the regularity conditions

$$T_i \rightarrow 0, \quad i = p, d \quad \text{for } |z| \rightarrow \infty, \quad 0 \leq t \leq t_s. \quad (5)$$

The scheme of contacting bodies during braking is shown in Fig. 1. It is supposed that at the time $t = 0$ composite half-space is loaded by normal forces of intensity p pressed down to the disc. There is a heat generation due to the friction on the working surface, which leads to a heating of the friction couple. The intensity of friction heat flux $q(t)$ in Eq. (4) is equal to the rate of frictional heat generation [2]

$$q(t) = f p(t) V(t), \quad 0 \leq t \leq t_s. \quad (6)$$

In particular case it can be assumed that the loading monotone increases from zero at time $t = 0$ to the nominal value p_0 according to the relation [6]:

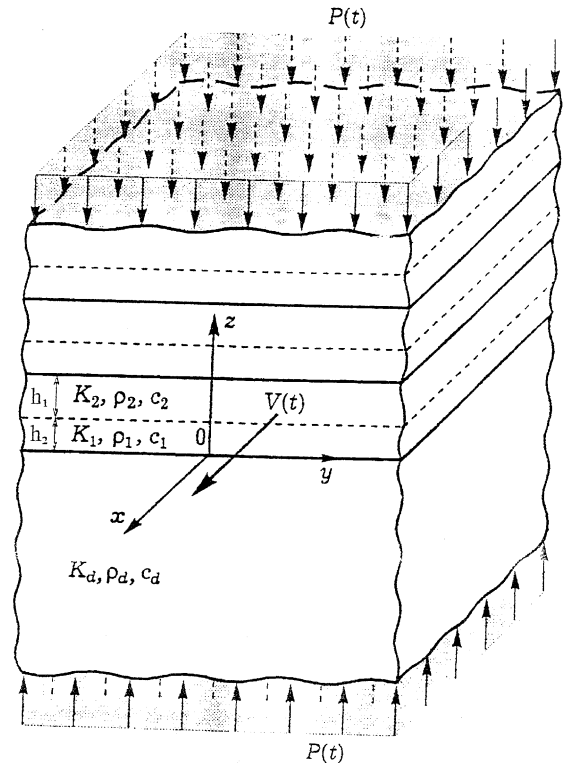


Fig. 1. Contact scheme of the frictional composite strap and homogeneous disc during braking.

$$p(t) = p_0 p^*(t), \quad (7)$$

$$p^*(t) = 1 - \exp(-t/t_m).$$

When braking a body on an horizontal plane and, considering only inertia effects, the speed during braking $V(t)$ changes from the initial value V_0 at $t = 0$ to zero at the moment of stopping $t = t_s$ as follows [6]:

$$V(t) = V_0 V^*(t), \tag{8}$$

$$V^*(t) = 1 - [t - t_m p^*(t/t_m)]/t_s^0, \quad 0 \leq t \leq t_s,$$

where $t_s^0 = 2W/fp_0V_0$ denotes the time of braking in the case of braking for very fast reaching of nominal value of loading p_0 ($t_m = 0$). In the case of $t_m \neq 0$, for finding of the time of stopping t_s the condition $V(t_s) = 0$ is applied and according to Eq. (8) it leads to the following nonlinear equation:

$$t_s - t_m p^*\left(\frac{t_s}{t_m}\right) = t_s^0, \quad 0 \leq t_m \leq t_s. \tag{9}$$

The half-space $z \geq 0$ is assumed to be composed of a periodic two-layered composite, in which the fundamental layer (lamina) contains two different layers of thicknesses h_1 and h_2 , respectively. Let h , $h = h_1 + h_2$ denote the thickness of the fundamental layer. The perfect mechanical and thermal contact between the layers being components of composite is assumed. The effective coefficients of heat conductivity K_p and thermal diffusivity k_p of the considered layered composite obtained by using the homogenization procedure with microlocal parameters [3] are the following form [7]:

$$K_p = K_1 \left(1 + \frac{[K]}{\eta \tilde{K}}\right), \quad k_p = \frac{K}{\tilde{c} \tilde{\rho}},$$

$$K = \tilde{K} - \frac{[K]^2}{\tilde{K}}, \quad \tilde{K} = \frac{K_1}{\eta} + \frac{K_2}{1 - \eta}, \tag{10}$$

$$(\tilde{K}, \tilde{\rho}, \tilde{c}) = \eta(K_1, \rho_1, c_1) + (1 - \eta)(K_2, \rho_2, c_2),$$

$$[K] = K_2 - K_1,$$

where

$$\eta = \frac{h_1}{h}.$$

It is assumed that the Archaid’s wear model on the contact surface [8] is given by

$$I(t) = \int_0^t m(T)q(t_0) dt_0, \quad 0 \leq t \leq t_s, \tag{11}$$

where $I(\cdot)$ is the total normal displacement of the working surface of the half-space, and the intensity of frictional heat flux q is determined by Eqs. (6)–(8), as well as the wear coefficient $m(\cdot)$ is linear function of the contact temperature

$$m(t) = m_0 + m_1 T(t). \tag{12}$$

The wear coefficient m_0 characterizes the wear due to contact load p and m_1 characterizes the wear caused by

contact temperature T . The linear dependence (12) is valid for small temperature gradients [9].

3. The temperature

The solution of the boundary value problem of heat conduction (1)–(5) obtained by using the integral Laplace transform with respect of time t can be written in the form of convolution integrals [10]

$$T_i(z, t) = A_0 A^* \int_0^\tau p^*[(\tau - \tau_0)/\tau_m] V^*(\tau - \tau_0) \tau_0^{-1/2} \times \exp(-\zeta_i^2/\tau_0) d\tau_0, \tag{13}$$

$$i = p, d, \quad 0 \leq \tau \leq \tau_s,$$

where

$$A_0 = \frac{fp_0V_0}{K_p} \sqrt{\frac{k_d t_s^0}{\pi}},$$

$$\tau = \frac{t}{t_s^0}, \quad \tau_m = \frac{t_m}{t_s^0}, \quad \tau_s = \frac{t_s}{t_s^0}, \tag{14}$$

$$A^* = \frac{1}{1 + k_e}, \quad k_e = \frac{K_p}{K_d} \sqrt{\frac{k_d}{k_p}},$$

$$\zeta_i = \frac{|z|}{2\sqrt{k_i t_s^0}}, \quad i = p, d.$$

Substituting functions $p^*(\cdot)$ and $V^*(\cdot)$ into the integral given in (13) and calculating for $\zeta_i = 0$, the temperature on the frictional surface is given in the form

$$T(t) = A_0 A^* T^*(t), \quad 0 \leq t \leq t_s, \tag{15}$$

where

$$T^*(t) = \left(2 + \tau_m - \frac{4}{3}\tau\right) \sqrt{\tau} - \left(1 + \frac{3}{2}\tau_m - \tau\right) \times 2\sqrt{\tau_m} F(\sqrt{\tau^*}) + \tau_m \sqrt{2\tau_m} F(\sqrt{2\tau^*}), \tag{16}$$

$$\tau^* = \frac{t}{t_m}$$

as well as

$$F(\tau) = \exp(-\tau^2) \int_0^\tau \exp(x^2) dx$$

is the Dousons integral which can be expressed as follows [11]:

$$F(\tau) = \sum_{i=0}^\infty \frac{(-2\tau^2)^i}{(2i+1)!!}, \quad 0 \leq \tau \leq 3,$$

$$(2i+1)!! = 1 \cdot 3 \cdot 5 \cdot \dots \cdot (2i+1),$$

$$F(\tau) = \sum_{i=0}^\infty \frac{(2i-1)!!}{(2\tau^2)^{i+1}}, \quad \tau > 3$$

and $(-1)!! = 1$.

For $t_m = 0$ ($t_s = t_s^0$) from Eq. (15) the relation for the contact temperature during braking with constant pressure during braking [12] can be written as

$$T(t) = 2A_0A^* \left(1 - \frac{2}{3} \frac{t}{t_s}\right) \sqrt{\frac{t}{t_s}}, \quad 0 \leq t \leq t_s.$$

4. Wear

By substitution of (15) into Eq. (11) and using (12) after some calculations we obtain the wear relation of the frictional surface of the composite half-space

$$I(t) = m_0^* I_0(t) + m_1^* A_0 A^* I_1(t), \quad 0 \leq t \leq t_s, \quad (17)$$

where

$$\begin{aligned} I_0(t) &= \tau - \tau^2/2 + \tau_m(\tau - \tau_m - 1)p^*(\tau^*) \\ &\quad + \tau_m^2 p^*(2\tau^*)/2, \\ I_1(t) &= I_1^{(1)}(t) + I_1^{(2)}(t) + I_1^{(3)}(t), \\ I_1^{(1)}(t) &= \frac{2}{3}(1 + \tau_m)(2 + \tau_m)\tau\sqrt{\tau} - \frac{2}{15}(10 + 7\tau_m)\tau^2\sqrt{\tau} \\ &\quad + \frac{8}{21}\tau^3\sqrt{\tau} - (1 + 2\tau_m)(2 + \tau_m) \\ &\quad \times \tau_m\sqrt{\tau_m^*} \left[\frac{1}{2}\sqrt{\pi}\operatorname{erf}(\sqrt{\tau^*}) - \tau^* \exp(-\tau^*) \right] \\ &\quad + \frac{1}{3}(10 + 11\tau_m)\tau_m^2\sqrt{\tau_m} \left[\frac{3}{4}\sqrt{\pi}\operatorname{erf}(\sqrt{\tau^*}) \right. \\ &\quad \left. - \sqrt{\tau^*} \left(\frac{3}{2} + \tau^*\right) \exp(-\tau^*) \right] \\ &\quad - \frac{4}{3}\tau_m^3\sqrt{\tau_m} \left[\frac{15}{8}\sqrt{\pi}\operatorname{erf}(\sqrt{\tau^*}) - \sqrt{\tau^*} \left(\frac{15}{4} + \frac{5}{2}\tau^* + \tau^{*2}\right) \right. \\ &\quad \left. \times \exp(-\tau^*) \right] + \frac{1}{2}(2 + \tau_m)\tau_m^2\sqrt{\tau_m} \left[\frac{1}{2}\sqrt{\frac{\pi}{2}}\operatorname{erf}(\sqrt{2\tau^*}) \right. \\ &\quad \left. - \sqrt{\tau^*} \exp(-\tau^*) \right] - \frac{3}{2}\tau_m^3\sqrt{\tau_m} \left[\frac{3}{8}\sqrt{\frac{\pi}{2}}\operatorname{erf}(\sqrt{2\tau^*}) \right. \\ &\quad \left. - \sqrt{\tau^*} \left(\frac{3}{8} + \tau^*\right) \exp(-\tau^*) \right], \\ I_1^{(2)}(t) &= 2\sqrt{\tau_m} \left[\left(2 + \frac{5}{2}\tau_m\right)M_{101}(\tau) - (1 + \tau_m) \left(1 + \frac{3}{2}\tau_m\right) \right. \\ &\quad \left. \times M_{001}(\tau) - M_{201}(\tau) + M_{211}(\tau) - \left(2 + \frac{7}{2}\tau_m\right) \right. \\ &\quad \left. \times M_{111}(\tau) + (1 + 2\tau_m) \left(1 + \frac{3}{2}\tau_m\right)M_{011}(\tau) \right. \\ &\quad \left. + \tau_m M_{121}(\tau) - \tau_m \left(1 + \frac{3}{2}\tau_m\right)M_{021}(\tau) \right], \\ I_1^{(3)}(t) &= \tau_m\sqrt{2\tau_m} \left[(1 + \tau_m)M_{002}(\tau) - M_{102}(\tau) + M_{112}(\tau) \right. \\ &\quad \left. - (1 + 2\tau_m)M_{012}(\tau) + \tau_m M_{022}(\tau) \right] \end{aligned}$$

and

$$\begin{aligned} M_{001}(\tau) &= \tau_m \left[\sqrt{\tau^*} - F(\sqrt{\tau^*}) \right], \\ M_{101}(\tau) &= \tau_m^2 \left[\sqrt{\tau^*} + \frac{1}{3}\tau^*\sqrt{\tau^*} - (1 + \tau^*)F(\sqrt{\tau^*}) \right], \\ M_{201}(\tau) &= \tau_m^3 \left[2\sqrt{\tau^*} + \frac{2}{3}\tau^*\sqrt{\tau^*} - \frac{1}{5}\tau^{*2}\sqrt{\tau^*} \right. \\ &\quad \left. - (2 + 2\tau^* + \tau^{*2})F(\sqrt{\tau^*}) \right], \\ M_{011}(\tau) &= \frac{\tau_m}{2} \left[\operatorname{erf}\sqrt{\tau^*} - \exp(-\tau^*)F(\sqrt{\tau^*}) \right], \\ M_{111}(\tau) &= \frac{\tau_m^2}{2} \left[\operatorname{erf}\sqrt{\tau^*} - \frac{1}{2}\sqrt{\tau^*} \exp(-\tau^*) \right. \\ &\quad \left. - \left(-\frac{1}{2} + \tau^*\right) \exp(-\tau^*)F(\sqrt{\tau^*}) \right], \\ M_{211}(\tau) &= \frac{\tau_m^3}{2} \left[\frac{7}{4}\operatorname{erf}\sqrt{\tau^*} - \frac{1}{2}\sqrt{\tau^*} \left(\frac{5}{2} + \tau^*\right) \exp(-\tau^*) \right. \\ &\quad \left. - \tau^{*2} \exp(-\tau^*)F(\sqrt{\tau^*}) \right. \\ &\quad \left. - \left(-\frac{1}{2} + \tau^*\right) \exp(-\tau^*)F(\sqrt{\tau^*}) \right], \\ M_{021}(\tau) &= \frac{\tau_m}{3} \left[\frac{1}{2}\sqrt{\frac{\pi}{2}}\operatorname{erf}(\sqrt{2\tau^*}) - \exp(-2\tau^*)F(\sqrt{2\tau^*}) \right], \\ M_{121}(\tau) &= \frac{\tau_m^2}{2} \left[\frac{1}{2}\sqrt{\frac{\pi}{2}}\operatorname{erf}(\sqrt{2\tau^*}) - \frac{1}{4}\sqrt{\tau^*} \exp(-2\tau^*) \right. \\ &\quad \left. - \left(\frac{1}{3} + \tau^*\right) \exp(-2\tau^*)F(\sqrt{2\tau^*}) \right], \\ M_{002}(\tau) &= \tau_m \left[\sqrt{\frac{\tau^*}{2}} - \frac{1}{2}F(\sqrt{2\tau^*}) \right], \\ M_{102}(\tau) &= \tau_m^2 \left[\frac{1}{2}\sqrt{\frac{\tau^*}{2}} + \frac{\tau^*}{3}\sqrt{\frac{\tau^*}{2}} - \frac{1}{2} \left(\frac{1}{2} + \tau^*\right)F(\sqrt{2\tau^*}) \right], \\ M_{112}(\tau) &= \tau_m^2 \left[\frac{5}{9\sqrt{2}}\operatorname{erf}(\sqrt{\tau^*}) - \frac{1}{3}\sqrt{\frac{\tau^*}{2}} \exp(-\tau^*) \right. \\ &\quad \left. - \frac{1}{3} \left(\frac{1}{3} + \tau^*\right) \exp(-\tau^*) - F(\sqrt{2\tau^*}) \right], \\ M_{012}(\tau) &= \tau_m \left[\frac{2}{3\sqrt{2}}\operatorname{erf}(\sqrt{\tau^*}) - \frac{1}{3} \exp(-\tau^*)F(\sqrt{2\tau^*}) \right], \\ M_{022}(\tau) &= \tau_m \left[\frac{\sqrt{\pi}}{8}\operatorname{erf}(\sqrt{2\tau^*}) - \frac{1}{4} \exp(-2\tau^*)F(\sqrt{2\tau^*}) \right] \end{aligned} \quad (18)$$

as well as

$$m_0^* = m_0 f V_0 p_0 t_s^0, \quad m_1^* = m_1 f V_0 p_0 t_s^0. \quad (19)$$

5. Numerical analysis

The calculations were performed according to the following scheme:

1. The dimensionless time τ_m of increasing loading from zero to the maximal value p_0 is given.

2. From Eq. (9) the dimensionless braking time τ_s is determined.
 3. By using Eq. (15) the temperature $T(\cdot)$ is obtained and by using (17)–(19), the wear $I_0(t)$ is calculated.
- It is shown from these relations that the influence of thermo-physical and geometrical parameters of the composite on the temperature and wear is determined the factor A^* (see Eq. (14)). The coefficient k_e , which defined parameter A^* , characterizes thermal activity of the composite body with respect to the homogeneous half-space [10]. Bearing in mind Eq. (10), the coefficient k_e can be expressed in the form

$$k_e = k_e^* g_p(K^*) \sqrt{\frac{\tilde{g}(p^*) \tilde{g}(c^*)}{g(K^*)}}, \quad (20)$$

where

$$K^* = \frac{K_2}{K_1}, \quad \rho^* = \frac{\rho_2}{\rho_1}, \quad c^* = \frac{c_2}{c_1},$$

$$k_e^* = \sqrt{\frac{K_1 \rho_1 c_1}{K_d \rho_d c_d}},$$

$$g(x) = \tilde{g}(x) - \frac{[g(x)]^2}{\hat{g}(x)},$$

$$g_p(x) = 1 + \frac{[g(x)]}{\eta \hat{g}(x)},$$

$$\hat{g}(x) = \frac{1}{\eta} + \frac{x}{1 - \eta},$$

$$\tilde{g}(x) = \eta + (1 - \eta)x, \quad [g(x)] = x - 1.$$

Thus, the input parameters for calculation of A^* are K^* , ρ^* , c^* , η and k_e^* .

It can be observed that an increasing of the conductivity ratio K^* leads to a decreasing of A^* (Fig. 2). If the thickness h_1 of one of the composite components increases then the ratio A^* decreases for $K^* < 1$ and grows for $K^* > 1$ (Fig. 3). For $c^* < 1$ an increasing of the parameter η leads to a decreasing of A^* but for $c^* > 1$ it leads to an increasing of A^* (Fig. 4).

The dimensionless temperature $T^*(\cdot)$ achieves maximal values for braking with a constant pressure during braking (Fig. 5). The distribution of $T^*(\cdot)$ is characterized by a considerable variability. The function $T^*(\cdot)$ increases fast at initial times to the maximal values and decreases with $t \rightarrow t_s$.

The dimensionless function $I_0(t)$ given in (17) characterizes the abrasive wear of working surface due to mechanical load, (7). The function $I_0(t)$ archives its maximal values at the stopping moment (Fig. 6). The most intensive wear is observed during braking with a constant pressure. The wear is independent on the parameter τ_m at the stopping moment.

The dimensionless function $I_1(t)$ given in (18) characterizes the influence of the contact temperature and it monotone increases, see Fig. 7. However its maximal

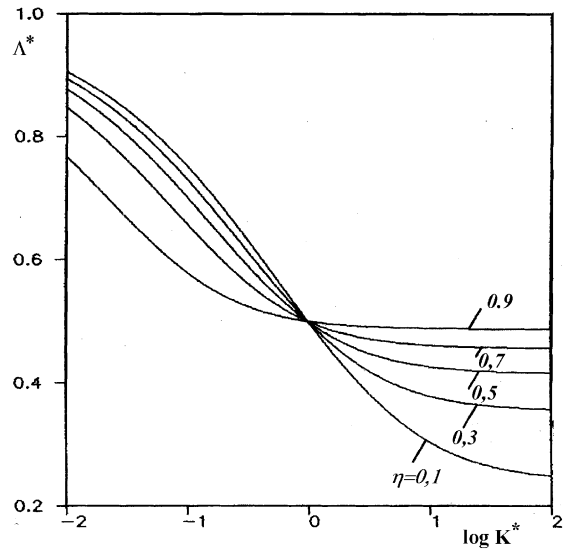


Fig. 2. Dependence of the dimensionless factor A^* on the ratio K^* at $k_e^* = 1$; $\rho^* = 1$; $c^* = 1$ for different values of the dimensionless thickness η .

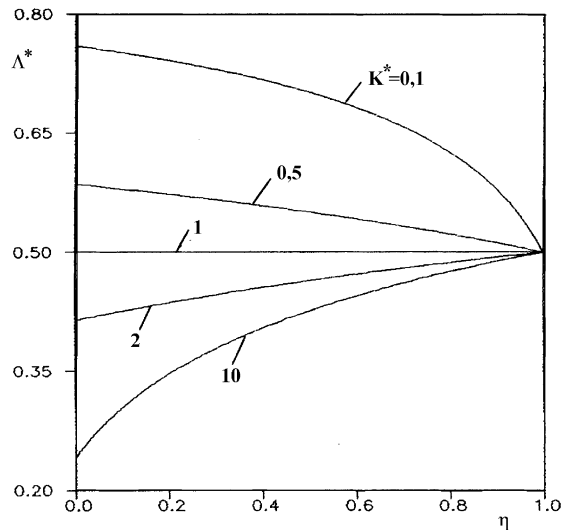


Fig. 3. Dependence of the dimensionless factor A^* on the dimensionless thickness η at $k_e^* = 1$; $\rho^* = 1$; $c^* = 1$ for different values of the ratio K^* .

value is reached at the stopping moment for $t = t_s$ and it is dependent on the parameter τ_m . For the fixed time of braking, the function $I_1(t)$ takes the minimal value in the case of constant pressure.

The ratio A^* is linearly represented in the relations for temperature (see, Eq. (15)) and for wear (see, Eq. (17)), so the dependence of temperature and wear on the effective properties of composite is determined by A^* .

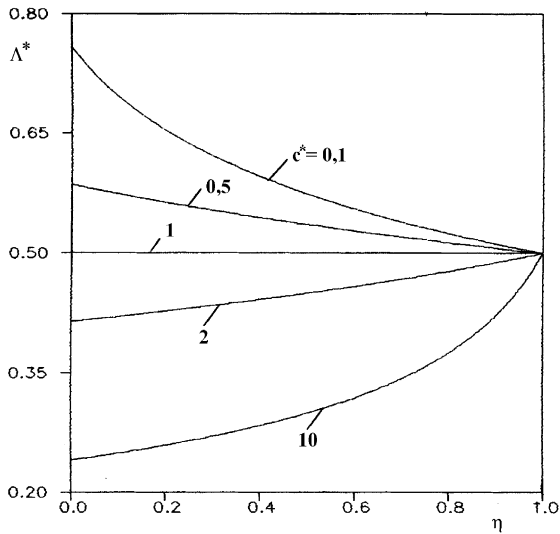


Fig. 4. Dependence of the dimensionless factor A^* on the dimensionless thickness η at $k_g^* = 1$; $\rho^* = 1$; $K^* = 1$ for different values of the ratio c^* .

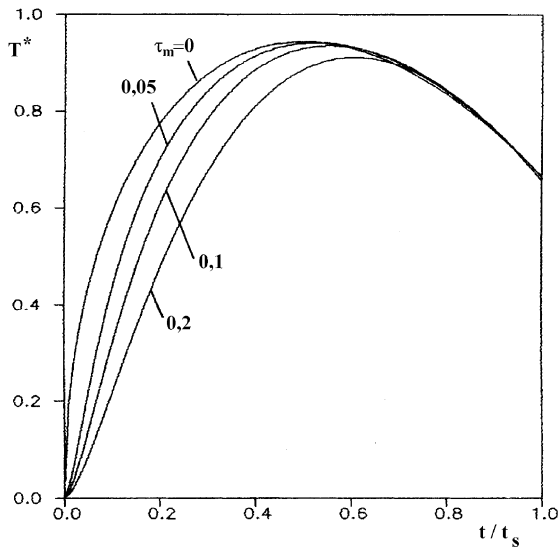


Fig. 5. Distribution of the dimensionless contact temperature $T^* = T/(A_0 A^*)$ during braking for different values of the dimensionless parameter τ_m .

6. Conclusions

It has been established that:

1. The increasing of content of the composite component with a greater coefficient of thermal conductivity ($K_1 > K_2$) in the fundamental lamina leads to a decreasing of the contact temperature. On the contrary,

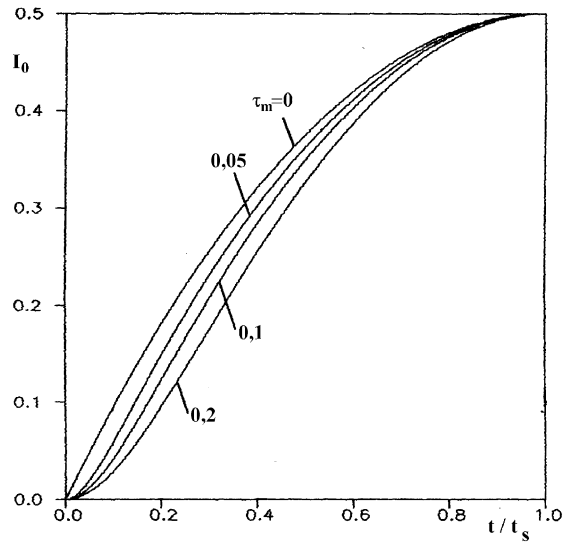


Fig. 6. Distribution of the dimensionless function I_0 during braking for different values of the dimensionless parameter τ_m .

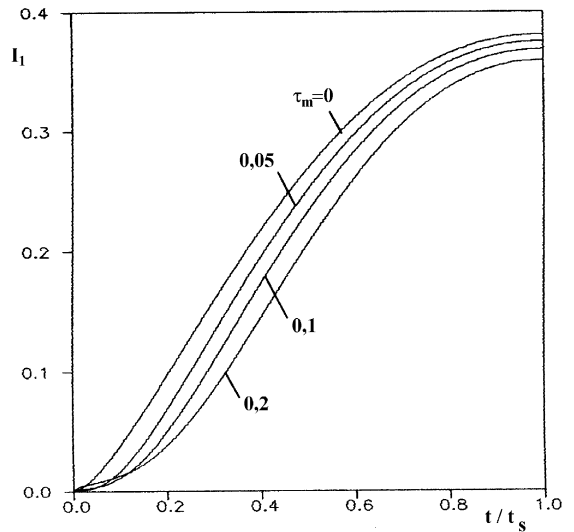


Fig. 7. Distribution of the dimensionless function I_1 during braking for different values of the dimensionless parameter τ_m .

the temperature on the frictional surface during braking will be the highest for greater content of thermoinsulator (the material with small coefficient of thermal conductivity).

2. The increasing of thickness of the composite component with great coefficient of specific heat ($c_1 > c_2$) leads also to a decreasing of the contact temperature. Thus, the temperature in the contact region during braking will be decreased, when in the composite half-space contents more material with greater coefficients of thermal conductivity and specific heat.

3. For the considered scheme of loading the wear reached the maximal value at the stop time t_s . The thermal part of the wear denoted by I_1 is considerably dependent on the time t_m of the duration of the action of loading from zero to the maximal value and the mechanical part of the wear denoted by I_0 is not dependent on the time t_m .

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